MA / MSC MT - 05

December - Examination 2015

M.A. / MSc. Previous Mathematics Examination

Mechanics

Paper - MA / MSC MT - 05

Time : 3 Hours] **[Max. Marks :- 80**

Note : The question paper is divided into three sections A, B and C. Use of calculator is allowed in this paper.

Section - A

8 x 2 = 16

Note : Section 'A' contain 08 very short answer type questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit is 30 words.

1) (i) Define a doublet.

(ii) Write the equation of motion under impulsive forces in vector form.

(iii) Define the conservative field of force.

(iv) Write the equation of continuity in cylindrical polar coordinate.

(v) Define Laminar flow.

(vi) Write the principle of Least Action.

(vii) What is the degree of freedom for a single particle moving in space at any time?

(viii) State the principle of conservation of Linear momentum.

Section - B 4 x 8 = 32

Note : Section 'B' contain 8 short answer type questions. Examinees will have to answer any four 04 questions. Each question is of 08 marks Examinees have to delimit each answer in maximum 200 words.

2) Explain the theorem of Parallel axes for moment of Inertia and for product of Inertia.

3) A uniform solid cylinder is placed with its axis horizontal on a plane. Whose inclination to the horizon is α. Show that the least coefficient of friction between it and the plane, so that is may roll and not slide is $\frac{1}{3}\tan\alpha$.

4) Prove that for a rigid body moving about a fixed point $\frac{dT}{dt} = \vec{W} \cdot \vec{G}$ where G is the moment of external forces about fixed point and $T = \frac{1}{2}\vec{H} \cdot \vec{W}$, where H is the angular momentum about the fixed point.

5) A dice in the form of a portion of parabola bounded by its latus rectum and its axis has its vertex A fixed end is stuck by a blow through the end of its latus rectum perpendicular to its plane. Show that the dice starts revolving about a line through A inclined at an angle $\tan^{-1}\left(\frac{14}{25}\right)$ to the axis.

6) A uniform circular board of mass M and radius a, is placed on a perfectly smooth horizontal plane and free to rotate about a vertical axis through its centre; a man of mass M′, walks round the edge of the board whose upper surface is rough enough to prevent his slipping: when he has walked completely round the board, to his starting point, show that board has turned through an angle $\frac{M'}{M+2M'}4\pi$

7) Find the stability condition for the motion of a top when the axis of top is not vertical.

8) Give $u=-Wy$, $v=Wx$ and $w=0$ show that the surfaces intersecting the stream line orthogonally exist and are the planes through z-axis.

9) What arrangement of sources and sinks will give rise to the function? $w=\log\left(z-\frac{a^2}{z}\right)$?

Section - C 2 x 16 = 32

Note : Section ‘C’ contain 04 Long answer type questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) (i) If σ is the cross sectional area of a stream filament, establish the equation of continuity in the form $\frac{\partial}{\partial t}(p\sigma)+\frac{\partial}{\partial s}(P\sigma q)=0$ where s is measured along the filament in the direction of flow and q is the speed.

(ii) Steam is rushing from a boiler through a conical pipe the diameter of the ends of which are D and d. If V and v be the corresponding velocities of the steam and if the motion be supposed to be that of divergence from the vertex of the cone. Prove that $\frac{v}{V} = \frac{D^2}{d^2} e^{(v^2 - V^2)/2k}$.

Where k is the pressure divided by the density and supposed constant.

11) Show that the Hamilton's principle function S for simple harmonic motion in a straight line is

$$\frac{\sqrt{\mu}\,(x^2 + x_0^2) \cos (t - t_0) \sqrt{\mu} - 2xx_0}{2 \sin (t - t_0) \sqrt{\mu}}$$

Where x, x_0 are the displacements from the centre of force at time t, t_0 respectively.

12) Two equal rods AB and BC each of length I, smoothly joined at B are suspended from A and oscillates in a vertical plane through A. Show that the periods of normal oscillations are $\frac{2\pi}{n}$ where $n^2 = \left(3 \pm \frac{6}{\sqrt{7}}\right) \frac{g}{l}$

13) Discuss the motion of a uniform sphere which rolls down an inclined plane, and rough enough to prevent any slipping.